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Multiwavelength laser exposures pose unique safety challenges for both laser experts and users of laser systems. We describe two methods of optimizing optical density requirements for multiwavelength laser exposures. The problem is first formulated and solved using standard mathematical programming techniques, and the results are compared to those from a simplified algorithm. The result is a method that can be efficiently integrated into existing laser modeling and hazard analysis software. As an example, overall visible light transmittance is maximized while maintaining eye-safe viewing conditions during a multiwavelength exposure, simultaneously minimizing the total optical density required for sufficient laser eye protection. This optimization formulation helps laser system users determine the proper laser eye protection for safe viewing in multiwavelength laser environments. © *2006 Laser Institute of America.*

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Optimization of optical density requirements for multiwavelength laser safety

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Multiwavelength laser exposures pose unique safety challenges for both laser experts and users of laser systems. We describe two methods of optimizing optical density requirements for multiwavelength laser exposures. The problem is first formulated and solved using standard mathematical programming techniques, and the results are compared to those from a simplified algorithm. The result is a method that can be efficiently integrated into existing laser modeling and hazard analysis software. As an example, overall visible light transmittance is maximized while maintaining eye-safe viewing conditions during a multiwavelength exposure, simultaneously minimizing the total optical density required for sufficient laser eye protection. This optimization formulation helps laser system users determine the proper laser eye protection for safe viewing in multiwavelength laser environments.

Key words: optimization, nonlinear programming, mathematical programming, laser safety, optical density (OD), multiple mode lasers, multiwavelength lasers

I. INTRODUCTION

A multiwavelength laser can be defined as a laser system emitting more than one discrete wavelength simultaneously. If each wavelength is considered a separate "mode" of operation, then each of these modes may have differing beam diameters, divergence values, or even time-intensity variations. In some instances these multiwavelength systems are the result of gas lasers operating in "all-line" configurations, or pulsed lasers equipped with nonlinear optical devices which may produce fundamental, frequency-doubled and frequency-tripled output.

Safety parameters for multiwavelength lasers cannot be computed by using only a single analytical mathematical formula. The overall exposure is increased compared to that when only a single wavelength is emitted at a time, and relative exposure magnitudes may vary as a function of exposure distance or time. This prevents applying the common equations[1] for single-wavelength exposures to a multiwavelength laser problem. While previous work by Lyon[2] has described the basic methodology required to provide an approach to safety, it stops short of addressing all considerations. This article extends the methods of Lyon[2] to determine optimal laser eye protection (LEP) design, which both minimizes overall attenuation and provides for optimization of user performance through the maximization of other metrics such as visible light transmittance.

Single-wavelength hazard analysis requires finding maximum permissible exposure (MPE) limits, accessible emission limits (AEL), and the effective amount of energy passing through the limiting aperture for each mode (Q_f). This article assumes that the reader is familiar with the techniques[3] for determining these parameters according to the ANSI Z136.1-2000 (Ref. 1) Safety Standard and has a basic understanding of laser safety analysis methods and terminology. Similar constructs and methods are used as part of the process outlined here for multiwavelength analysis.

Here, we examine two approaches to the estimation of optical density (OD) requirements in a multiwavelength laser exposure. The first approach formulates the problem at a relatively high level using mathematical programming, formulated as a nonlinear optimization algorithm, and solved using GAMS,[4] a modeling language for representing the mathematical model. The second method accomplishes these goals through the development of a simplified algorithm, which we have implemented in the C++programming language[5] for the purposes of presenting comparative results.

II. PROBLEM DESCRIPTION

As described in Lyon,[2] when one is exposed to a laser emitting multiple wavelengths simultaneously, the combined effect should be considered. In order to denote the various parameters required for the analyses, we will adopt a convention which is similar to the ANSI Z136.1 Standard. Let $\lambda \in \mathrm{SM}$ (subset of modes) with the individual wavelengths emitted by the laser indexed by the symbol λ. The symbol $Q_f(\lambda)$ denotes the energy (in Joules) delivered through the corresponding limiting aperture at wavelength $\lambda \in \mathrm{SM}$, and $Q_{\mathrm{AEL}}(\lambda)$ denotes the accessible emission limit (in Joules) for wavelength $\lambda \in \mathrm{SM}$. This accessible emission limit is computed from the product of the MPE, and the area of the appropriate limiting aperture. Our interpretation of the work

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by Lyon[2] is that the sum of the ratios of exposures to exposure limits, for each wavelength or exposure condition considered is given later in Eq. (1). We translate the MPE representation [Eq. (1) of Ref. 2] to an expression which more accurately contains values averaged over limiting apertures. In absence of LEP, the safe exposure limit is considered to be exceeded if

$$\sum_{\lambda \in \mathrm{SM}} \frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)} > 1. \tag{1}$$

To avoid the situation described in Eq. (1), LEP is used, and in determining optimal LEP we effectively select an optical density $D(\lambda)$ at each wavelength $\lambda \in \mathrm{SM}$, which in turn yields a transmittance $\tau(\lambda)$ via $\tau(\lambda)=10^{-D(\lambda)}$, $\lambda \in \mathrm{SM}$. To ensure a safe level of exposure we require

$$\sum_{\lambda \in \mathrm{SM}} \left(\frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)}\right)\tau(\lambda) \leqslant 1. \tag{2}$$

Equation (2) will form one of the constraints used in the mathematical program used to optimally design LEP. The goal is to provide various criteria by which to optimize the LEP, while providing safe exposure to the multiwavelength laser. One option is to maximize visibility through the eyewear, and for that we require a scalar performance measure that captures *quality of vision* as a function of transmittance at each wavelength. The *spectral luminous efficiency* assigns a weight to each wavelength in the spectrum that captures its relative contribution to visibility in a typical environment for a user.[6]

There are, theoretically, infinite combinations of OD for each multiwavelength laser that would provide a solution to Eq. (2). There may be some modes that are already considered safe individually, i.e., $[Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)] \leqslant 1$. For the other modes, the OD applied for that mode has a minimum such that $10^{-D(\lambda)}[Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)]=1$. Beyond this, there are no constraints on how we apply optical densities.

From a more practical standpoint, there may be other considerations such as the cost involved in producing OD values for specific wavelengths or the desire to maximize transmittance over some wavelength band. In this case, we may correspondingly assign a positive weighting value to each mode $V(\lambda)$, $\lambda \in \Lambda$. If we are not concerned with discriminating between the wavelengths being emitted, a uniform weight of 1.0 may be applied for each $V(\lambda)$, which would in turn simply minimize the sum of the optical densities for each mode.

III. EVALUATION PARAMETERS

Several parameters must first be gathered in order to determine the optimal OD requirement for each mode of a multiwavelength laser exposure.

$Q_f(\lambda)$—The total amount of energy expected to enter the applicable limiting aperture for each mode. This value takes all modifiers into account besides OD, including but not limited to atmospheric attenuation, transmittance and magnification inherent to aided viewing scenarios, beam spread due to laser geometry, exposure duration, etc. The ANSI Z136.1-2000 (Ref. 1) provides guidelines and calculations for finding this value given a particular or a worst-case scenario.

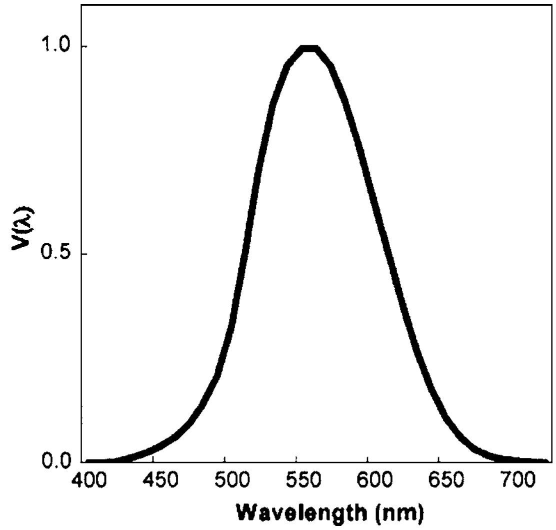

FIG. 1. Visible luminous efficiency for the human eye.

$Q_{\mathrm{AEL}}(\lambda)$—The accessible emission limit, or desired energy exposure considered to make this particular mode of the laser safe. The ANSI Z136.1-2000 (Ref. 1) may be used to find the MPE and use that to determine the worst-case AEL for the scenario. A less conservative value may also be used here depending on the user's needs.

The values for $Q_{\mathrm{AEL}}(\lambda)$ and $Q_f(\lambda)$ may have been determined in watts, joules, or some other related unit of power or energy. The units used are not important, so long as the units used are consistent between $Q_{\mathrm{AEL}}(\lambda)$ and $Q_f(\lambda)$ such that the ratio between them, $[Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)]$, is meaningful.

$V(\lambda)$—A relative weight for each wavelength that will help to discriminate the assignment of OD values. This value must be greater than zero at each wavelength, and must be constant with respect to the OD assigned. Modes with a relatively higher $V(\lambda)$ will be assigned a lower OD such as to maintain the objective function, Eq. (3). Regardless of this weight, the final solution will always provide a safe exposure limit if the wavelength were considered independently, i.e., $10^{-D(\lambda)}[Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)] \leqslant 1$. If we are not concerned with discriminating between the modes, a weight of 1.0 may be applied for each $V(\lambda)$, which would simply provide a solution that minimizes the sum of the optical densities for each wavelength.

An example weighting value $V(\lambda)$, which represents the *spectral luminous efficiency* curve shown in Fig. 1 as a function of wavelength,[6,7] can be used to maximize visibility. Because the human eye senses different colors (wavelengths) with varying acuity, when the wavelengths are outside the visible spectrum, visual acuity is effectively zero, but peaks around 550 nm. In our research, we have the option of weighting the transmittance, $\tau(\lambda)$, at each wavelength by $V(\lambda)$ and therefore maximize visibility. We note that if these values are used, it is important to provide some minimal value for wavelengths on the "wings" of the curve that is greater than zero. The examples used in the article will use values from $V(\lambda)$ shown in Fig. 1.

IV. PROCEDURE FOR OPTICAL DENSITY DETERMINATION

Optimal design of LEP for multiwavelength lasers begins with the same calculations and requirements as outlined in the ANSI Z136.1-2000 (Ref. 1) for single-wavelength exposures. However, as previously indicated, when laser exposures involve more than one wavelength, the calculations are no longer straightforward, and so, in this paper mathematical programming and algorithmic approaches are used to determine optimal LEP design.

Our assumption is that we are considering wavelengths which are all in a "retinal hazard" region of the spectrum ($0.400-1.400\ \mu$m) as described by the exposure limits of the ANSI Z136.1-2000 Standard.[1] However, the method can be applied to any analysis of common exposure site hazard bands such as a number of wavelengths which are all cornea hazards. We also recognize that when carried to extreme precision, the algorithm can result in physically unnecessary iterations, resulting in very small optimizations in optical density values. We ask the reader to recognize that the algorithm presented is accurate although it does not examine precision of optical density values in the same way that a trained laser safety professional might see a reasonable exit criterion based upon available laser eye protection solutions.

A. Mathematical programming approach

Mathematical programming is a set of models, methods and theory that deals with optimization of typically complex systems. The field includes linear programming, nonlinear programming, and integer programming. It primarily concerns minimizing or maximizing an objective function subject to one or more constraints. Generally, mathematical programs seek to optimize some objective through the selection of a set of values for the decision variables.[8] These variables are constrained by various conditions and restrictions. Mathematical programming chooses levels of the decision variables so that the objective is optimized while maintaining the integrity of the constraints.

The problem of optimal LEP design is first formulated as a nonlinear optimization algorithm and solved using GAMS.[4] An objective function of this specific problem involves finding a solution which minimizes OD for each wavelength and maximizes visibility, while maintaining the constraint ensuring a safe exposure scenario. Although the logarithmic (not linear) relationship between OD and transmittance appears to cause significant difficulties, the specifics of this problem alleviate this complexity.

The problem was first formulated as a mathematical programming model. Because the constraint that ensures a safe viewing condition, Inequality (4), is nonlinear in form, the formulation is considered a nonlinear program. The decision variable OD(λ) is optimized according to the following model formulation:

$$\text{minimize} \quad Z = \sum_{\lambda \in \Lambda} D(\lambda) V(\lambda) \tag{3}$$

$$\text{subject to} \quad \sum_{\lambda \in \mathrm{SM}} \left(\frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)} \right) 10^{-D(\lambda)} \leqslant 1 \tag{4}$$

$$\text{and} \quad D(\lambda) \geqslant 0, \lambda \in \Lambda. \tag{5}$$

The objective function, Eq. (3), seeks to optimize the OD according the user-specified weighting values for $V(\lambda)$. When $V(\lambda)$ is the spectral luminous efficiency, the visibility of the LEP is maximized because the spectral luminous efficiency data, $V(\lambda)$, positively weights transmittance $\tau(\lambda) = 10^{-D(\lambda)}$ over the visible wavelengths. In this situation, the objective function prefers lower optical densities in the visible wavelengths and therefore more visible light will be transmitted through the LEP to the user's eye.

Inequality (4) is the constraint that ensures the overall exposure is safe. As described previously, a multiwavelength laser exposure is safe when Inequality (4) holds for the wavelengths being emitted ($\mathrm{SM} \subset \Lambda$). When multiplied by the transmittance $\tau(\lambda) = 10^{-D(\lambda)}$, the ratio of total energy delivered to the accessible emission limit must be less than 1. It is through this constraint that we have flexibility in setting the optical densities. For example, when maximizing visibility we can compensate for lower optical densities in the visible spectrum with higher optical densities in the nonvisible, or out-of-band wavelengths.

GAMS (Ref. 4) was configured using the earlier formulation and run against a number of representative laser exposures. The results of the analysis and their validation is described in Sec. V.

B. Algorithmic approach

In an effort to utilize the mathematical analysis formulated and run in GAMS,[4] an algorithm was independently formulated for the specific problem of hazard analysis. This algorithm is sufficiently simple such that an analysis can be conducted with a handheld calculator, or with basic programming skills can be implemented for use in software applications. The algorithm is limited in scope when compared to the capabilities of GAMS, but should suffice for most laser safety analysis requirements. It is instructive to examine the development of our algorithm through the use of an illustrative example. Working through the exercise provides insight to the logic applied to derive the final equations of the method.

After assembling all of the output parameters of the laser in question, the initial step in the algorithm is to determine if the laser is already safe without any additional OD from LEP. This involves simply summing up the ratios of each $[Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)]$ and checking to see if the sum is less than or equal to one, i.e., $\Sigma_\lambda [Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)] \leqslant 1$. The remainder of the procedure assumes that this is not the case. The goal of this algorithm then is to fulfill (3) and (4), restated as

$$A = \sum_{\lambda} 10^{-D(\lambda)} \frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)} = 1, \text{ while minimizing}$$

$$Z = \sum_{\lambda} D(\lambda) V(\lambda).$$

TABLE I. Initial algorithm values for example laser C.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V
458	0.02	$6.50E-04$	$5.56E-02$
488	0.1	$6.50E-04$	$1.94E-01$
514	0.2	$6.50E-04$	$5.86E-01$
640	0.05	$6.50E-04$	$1.75E-01$

If we consider these equations for each mode (wavelength) independently and take the first derivative of each with respect to $D(\lambda)$, we can easily see the effect of increasing the OD for each mode

$$A(\lambda) = 10^{-D(\lambda)}\frac{Q_f(\lambda)}{Q_{AEL}(\lambda)}$$

$$A'(\lambda) = 10^{-D(\lambda)}\frac{Q_f(\lambda)}{Q_{AEL}(\lambda)}[-\ln(10)], \tag{6}$$

$$Z(\lambda) = D(\lambda)V(\lambda),$$

$$Z'(\lambda) = V(\lambda). \tag{7}$$

Therefore, our approach is to set the goal of the algorithm to increase OD on the mode that has the greatest impact in decreasing A to 1 while having the least impact in raising Z. To help us do that, we will introduce a weighting function that combines (6) and (7):

$$W[D(\lambda)] = \frac{A'(\lambda)}{Z'(\lambda)} = 10^{-D(\lambda)}\frac{Q_f(\lambda)}{Q_{AEL}(\lambda)V(\lambda)}[-\ln(10)]. \tag{8}$$

Since we are only using the function for comparison between modes, we can factor out the common constant elements to get

$$W[D(\lambda)] = 10^{-D(\lambda)}\frac{Q_f(\lambda)}{Q_{AEL}(\lambda)V(\lambda)}. \tag{9}$$

The most beneficial effect of adding small increments of OD to individual modes of the laser is adding OD to the mode with the greatest W value for its current $D(\lambda)$. To help illustrate this, we will use the following example of a multimode laser (this is the laser named "C" in the results section). We have also used the spectral luminous efficiency curve described above to obtain illustrative values for V. Table I illustrates the initial values for the algorithm using laser C.

Examination of Table I quickly shows that each of the modes of this laser is hazardous ($Q_f/Q_{AEL}>1$ for each mode), so each mode has a minimal OD value (D_{min}) greater than zero. The values of D_{min} for each mode are shown in Table II.

Applying a minimal OD of $D(\lambda)$ from Table II makes each mode safe [$A(\lambda)$ for each mode is 1], but the laser as a whole is still not safe because there are four modes. We see that Eq. (6) would yield a value of 4 in this case. We still need to increase the OD for one or more modes. We can choose the mode to work on by computing $W(D)$.

If we solve for $W(D_{min})$ for each mode, we can find the mode with the greatest W that will allow us to increase its OD with the greatest impact on A and the least impact on Z. Table II illustrates the computation of W for each mode based upon the initial hazard analysis.

From the values in Table II, we see that the mode to address is the one with the highest W value based on D_{min}. Raising the OD for the 458-nm mode a small amount will have the greatest impact on A and the least impact on T. But how much can we raise this OD? Looking at Eq. (9), we will note that W is monotonically decreasing as D increases for all values that are greater than or equal to 0. This means continually raising the OD for one mode will have a decreasingly beneficial effect. We can increase the OD of the 458-nm mode until $W(D)$ is equal to the next lowest W value; in this case, the 640-nm mode has a W value of 5.71.

Using Eq. (9) and solving for $D(\lambda)$ we obtain Eq. (10):

$$D(\lambda) = -\log\left[\frac{W(\lambda)Q_{AEL}(\lambda)V(\lambda)}{Q_f(\lambda)}\right]. \tag{10}$$

In this case, we get a new value of $D(\lambda)$ for the 458-nm mode of 1.99. We now compute new values of A and Z to determine if the new combination of OD values will make the entire laser safe. Table III represents an updated configuration of parameters, along with the values of A and Z.

Unfortunately, this still does not yet provide for a safe exposure ($A>1$), so we need to continue. We can now look to the next highest W value, and increase the OD at the 458-nm *and* the 640-nm modes to the point where their W values are the same. In this case, the 488-nm mode has a W value of 5.15. We can apply (10) to the 458-nm *and* 540-nm modes to find new $D(\lambda)$ values for both of them. The results are shown in Table IV. We see that the value of A is indeed reduced, indicating that the exposure is less hazardous. However, A remains greater than 1, indicating that a hazard does indeed remain.

TABLE II. Individual mode hazard analysis values of D_{min} and weighting factors for example laser C.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	T	D_{min}	W
458	0.02	$6.50E-04$	$5.56E-02$	$3.25E-02$	1.49	18.0
488	0.10	$6.50E-04$	$1.94E-01$	$6.50E-03$	2.19	5.15
514	0.20	$6.50E-04$	$5.86E-01$	$3.25E-03$	2.49	1.71
640	0.05	$6.50E-04$	$1.75E-01$	$1.30E-02$	1.89	5.71

TABLE III. Individual mode hazard analysis values of $D_{\min}$ and weighting factors for example laser C after one algorithm iteration. $A=3.3177$, $Z=2.3288$.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D(\lambda)$	W
458	0.02	$6.50E-04$	$5.56E-02$	1.99	5.71
488	0.1	$6.50E-04$	$1.94E-04$	2.19	5.15
514	0.2	$6.50E-04$	$5.86E-01$	2.49	1.71
640	0.05	$6.50E-04$	$1.75E-01$	1.89	5.71

At every stage of iteration, we are using W to find the optimal mode and only increasing the OD for that mode which has the greatest impact in decreasing A with the least impact on increasing Z. We could continue with this method until we reach $A=1$.

We see that if we had to increase the OD for every mode, then every mode would eventually share the same W value. This indicates that we can solve for this W value (W') from the very beginning and use Eq. (10) to find the OD that would provide that W' value. Using $A=\Sigma_\lambda 10^{-D(\lambda)}Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)=1$ and replacing OD using (10) we get

$$\sum_\lambda 10^{\log[W' Q_{\mathrm{AEL}}(\lambda)V(\lambda)/Q_f(\lambda)]}\frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)}\lambda = 1. \tag{11}$$

This equation simplifies to

$$\sum_\lambda W' V(\lambda) = 1, \text{ or } W' \sum_\lambda V(\lambda) = 1 \tag{12}$$

for cases of constant values of W'. This indicates that we can again simplify to

$$W' = \frac{1}{\sum_\lambda V(\lambda)}. \tag{13}$$

So for our example laser C, we can find $W'=0.99$ from our values of $V(\lambda)$. This gives the resulting optical densities shown in Table V, and a safe multiwavelength laser exposure.

For a multimode laser where every mode is hazardous, the solution then is to find W' using Eq. (13), then find the value of $D(\lambda)$ for each mode using Eq. (10). If there are one or more modes that are *not* hazardous, we may not know if

TABLE IV. Individual mode hazard analysis values of $D_{\min}$ and weighting factors for example laser C after two algorithm iterations. $A=3.1874$, $Z=2.3332$.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D(\lambda)$	W
458	0.02	$6.50E-04$	$5.56E-02$	2.03	5.15
488	0.1	$6.50E-04$	$1.94E-01$	2.19	5.15
514	0.2	$6.50E-04$	$5.86E-01$	2.49	1.71
640	0.05	$6.50E-04$	$1.75E-01$	1.93	5.15

TABLE V. Individual mode hazard analysis values and weighting factors for example laser C applying Eq. (15) in the development of our algorithm. $A=1.0$, $Z=2.7762$.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D(\lambda)$	W
458	0.02	$6.50E-04$	$5.56E-02$	2.75	0.99
488	0.1	$6.50E-04$	$1.94E-01$	2.90	0.99
514	0.2	$6.50E-04$	$5.86E-01$	2.72	0.99
640	0.05	$6.50E-04$	$1.75E-01$	2.65	0.99

TABLE VI. Initial parameters for a modified laser C assessment of optical density requirements.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D_{\min}$	W
458	0.02	$6.50E-04$	$5.56E-02$	0	553.4
488	0.1	$6.50E-04$	$1.94E-01$	0	793.0
514	0.2	$6.50E-04$	$5.86E-01$	0	525.1
640	$6.0E-04$	$6.50E-04$	$1.75E-01$	0	5.3

TABLE VII. Algorithm results for assessment of a modified laser C. $A=1.0$, $Z=2.4397$.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D(\lambda)$	W
458	0.02	$6.50E-04$	$5.56E-02$	2.75	0.99
488	0.1	$6.50E-04$	$1.94E-01$	2.90	0.99
514	0.2	$6.50E-04$	$5.86E-01$	2.72	0.99
640	$6.00E-04$	$6.50E-04$	$1.75E-01$	0.73	0.99

those modes require any OD as part of the safety analysis. As an example of this condition, we take the example laser C and change the value of Q_f for the 640-nm mode to be nonhazardous and evaluate the optimal OD requirements. We begin with the parameters in Table VI. We can assume an initial OD of zero for each mode and use Eq. (9) to solve for the W values for each mode. This analysis is also shown in Table VI.

In this situation, we will note that the $W(0)$ values for the mode are still greater than the W' value calculated using Eq. (13), which was 0.99. This means that even though the 640-nm mode is not hazardous, the optimal solution would still involve raising the OD of this mode some amount. Once again, we apply Eq. (10) to calculate the OD for each mode in order to provide for a safe exposure. We obtained the results summarized by Table VII.

If one or more of the $W(0)$ values for the mode are less than or equal to the W' value, then these modes do not need any OD to reach an optimal solution. When this occurs, it is appropriate to remove those modes from the equation and solve for a new W' value. This can be done until all of the remaining modes have a $W(0)$ value of greater than W'. As a final case study, we modify our example laser C to present the situation when no OD is required at one wavelength. Our initial parameters are presented in Table VIII.

In this example, the Q_f value for the 640-nm mode is lowered such that its W value is less than the 0.99 that was calculated for W'. This indicates that the optimal solution would not include any OD for this mode. We also cannot have a negative OD value, so we should effectively remove this mode from the computation.

First, we calculate the effect that this mode would have on A. In this case, $A(\lambda)=10^{-D(\lambda)}Q_f(\lambda)/Q_{\mathrm{AEL}}(\lambda)$ for the 640-nm mode, such that $A(\lambda)=(6.00E-5)/(6.50E-4)=0.092\,31$. We effectively want the sum of $A(\lambda)$ of the other three modes to make the difference between 1.0 and 0.0923, or 0.9077. Using this new value for our desired A', we will find the new optimal W' using a derivation of Eq. (13): $W'=0.907\,69/\Sigma_{\lambda\in\Lambda}V(\lambda)$ for all of the *remaining* modes. So in this case we have: $W'=1.086\,27$. We then use Eq. (10) to find the OD for each of the remaining modes, giving us the optimal solution shown in Table IX.

To recap, the procedure can be summarized as follows:

(1) Find $W(\lambda)$ for each mode of the laser, starting with an OD of 0 for each mode. Using the simplified Eq. (9):

$$W(\lambda)=\frac{Q_f(\lambda)}{Q_{\mathrm{AEL}}(\lambda)V(\lambda)}. \tag{14}$$

(2) Find the optimal W' for the laser using (13):

$$W'=\frac{1}{\sum_\lambda V(\lambda)}. \tag{15}$$

Compare W' to the $W(\lambda)$ for each mode. If there are any $W(\lambda)<W'$, these modes will not have any increase in $D(\lambda)$. Call this subset of Λ, J and remove them from the calculation of W using

$$W'=\frac{1-\sum_j \dfrac{Q_f(\lambda_j)}{Q_{\mathrm{AEL}}(\lambda_j)}}{\sum_k V(\lambda_k)-\sum_j V(\lambda_j)}. \tag{16}$$

If this new W' is still greater than $W(\lambda)$ of one of the remaining modes, remove that mode in the same way as the step above until there is some subset of the original modes where $W(\lambda)\geqslant W'$ for each mode.

(3) For every mode remaining, find the required optical density using

$$D(\lambda)=-\log\left[\frac{WQ_{\mathrm{AEL}}(\lambda)V(\lambda)}{Q_f(\lambda)}\right]. \tag{17}$$

We note that in case of equally weighted $V(\lambda)=1.0$, then the trivial solution of increasing each optical density by

TABLE VIII. Initial parameters and for a second modified laser C assessment of optical density requirements. $A=1.0$, $Z=2.2760$.

Wavelength (nm)	Φ_f (W)	Φ_{AEL} (W)	V	$D_{\min}$	W
458	0.02	$6.50E-04$	$5.56E-02$	2.71	1.08627
488	0.1	$6.50E-04$	$1.94E-01$	2.86	1.08627
514	0.2	$6.50E-04$	$5.86E-01$	2.68	1.08627
640	$6.00E-05$	$6.50E-04$	$1.75E-01$	0	N/A

TABLE IX. Summary of data comparing mathematical programming results and the algorithm for five sample lasers.

Laser input parameters				Optical density results	
				Nonlinear programming	Algorithmic approach
"A"	Q_f (J)	Q_{AEL} (J)			
532 nm	1.00E−03	5.00E−07		3.301	3.302
1064 nm	5.00E−04	5.00E−06		10.000	4.945
			Safety check	*1.000*	*0.999*
"B"	Q_f (J)	Q_{AEL} (J)			
532 nm	1.00E−01	6.50E−04		2.187	2.188
860 nm	5.00E−01	1.40E−03		10.000	5.498
			Safety check	*1.000*	*0.999*
"C"	Q_f (J)	Q_{AEL} (J)			
458 nm	2.00E−02	6.50E−04		2.748	2.748
488 nm	1.00E−01	6.50E−04		2.903	2.903
514 nm	2.00E−01	6.50E−04		2.725	2.725
640 nm	5.00E−02	6.50E−04		2.648	2.648
			Safety check	*1.000*	*1.000*
"D"	Q_f (J)	Q_{AEL} (J)			
430 nm	1.00E−02	1.00E−05		4.829	4.829
530 nm	5.00E−04	6.50E−04		0.000	0.000
640 nm	5.00E−02	6.50E−04		2.552	2.552
			Safety check	*1.000*	*1.000*
"E"	Q_f (J)	Q_{AEL} (J)			
800 nm	1.00E−02	1.60E−02		2.085	0.097
1064 nm	1.00E−03	5.00E−06		2.303	2.602
			Safety check	*1.000*	*1.000*

$\log_{10}(N)$, where N is the number of individual laser modes.

V. DISCUSSION

The two methods were checked for agreement and accuracy using data from five example lasers; the results are shown in Table IX. The first column of this table contains the wavelengths that each example laser emits. The second and third columns are the $Q_f(\lambda)$ and $Q_{AEL}(\lambda)$ as calculated according to the ANSI Z136.1-2000.[1] The resulting optimal optical densities at each wavelength are shown for the two methods: nonlinear programming and the algorithmic approach in the fourth and fifth columns. The safety check is a manual calculation of the value of the constraint required for a safe exposure from Inequality (2). Recall that the value of the safety constraint, Inequality (2), must be less than or equal to 1 in order to ensure a safe exposure.

The first results column presented in Table IX are based on solving the nonlinear program using the MINOS NLP solver, which we call from the GAMS mathematical programming language.[9,4] These are compared to the results from the algorithmic approach implemented in the C++ programming language.[5,10] We see by the overly safe OD of 10 that resulted in a few of the examples (with the nonlinear mathematical program) that this particular formulation is not the best for that method, but provides a basis and comparison for the algorithmic approach.

By comparing the optical densities determined to be optimal for each example laser, it is clear that the wavelengths in the visible spectrum are being favored according to the weights set by the spectral luminous efficiency values. These weights prefer lower optical densities for the visible wavelengths and the models compensate by putting higher OD on the nonvisible wavelengths. This satisfies the goal of maximizing visibility while providing a safe viewing condition for multiwavelength exposures.

VI. CONCLUSIONS

This article presents a brief tutorial regarding the computation of optimal optical density values for multiwavelength lasers. It is our hope that with this information, hazard analyses may be conducted with more understanding and greater confidence in the results. The purpose of this work was to transition the optimization methods used in nonlinear programming into a versatile format that can be integrated into existing laser hazard software. It is recognized that the algorithm may produce iterations which might be considered "overly precise," but in the end are correct. A trained laser

safety professional will learn to recognize solutions which are within required precision after a few applications of the method, and may avoid unnecessary iterations.

The procedures detailed here are instructional and address several variations in laser parameters that can occur. The evaluation is simple to calculate by hand by anyone with a basic understanding of laser safety analysis.

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